Frequency analysis of hydrological extreme events and how to consider climate change

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1 INTRODUCTION

Using statistics, we have been regarding hydrological events as random variables and decided design rainfalls and design floods for various water-related facilities. Frequency of hydrological extreme events is therefore a concern of decision makers in practical works for water resources planning and hydraulic design of flood control facilities. This paper describes basic theory of hydrologic frequency analysis and statistics in terms of extreme events.

Recent years there are discussions how we can consider the climate change effect on hydrological time series, which has been regarded as stationary. However, if the global warming affects hydrological time series, we should treat it as non-stationary sequence. This paper also discusses this matter.

2 HYDROLOGIC FREQUENCY ANALYSIS

Hydrologic frequency analysis provides the basic information on planning, design and management of hydraulic and water resources systems, which are useful for saving our lives and properties in river basins. The significant issues that need to be addressed includes:

(1) data characteristics (homogeneity, independence),
(2) sample size (effect of years of records on accuracy and appropriate estimation method),
(3) parameter estimation (selection of parameter values of distribution functions),
(4) model evaluation (selection of a distribution), and
(5) accuracy of quantile estimates (unbiasedness, estimation error).

When dealing with samples of extreme events (datasets of storms, floods and droughts), we generally assume homogeneity and serial independence to make frequency analysis procedures straightforward. These assumptions are usually accepted since neither heterogeneity nor dependence is evidently recognized. Even if these assumptions are reasonable, the other four problems are still serious (Takara and Stedinger, 1994).

Many researchers have worked on these problems (Stedinger *et al.*, 1993). As a result, there are several options for estimation methods and choice of distribution. Practitioners now wonder which estimation method they should use and which is the best probability distribution to employ.

2.1 Distribution function, non-exceedance probability and frequency of extreme events

To describe the probability distribution of a random variable *X*, we use a cumulative distribution function, or CDF. The value of this function $F_X(x)$ is simply the probability *P* of the event that the random variable takes on value equal to or less than the argument:

$$F_X(x) = P[X \le x] \tag{1}$$

This is the probability that during the year the random variable in question, *X*, will not exceed some *x*; $F_X(x)$ is, therefore, regarded as *non-exceedance probability*. Hereafter, we use a simpler expression $F(x)$ for $F_X(x)$. The probability density function $f(x)$ of *X* is related to $F(x)$ as:

$$F(x) = \int_{-\infty}^{x} f(t)dt \tag{2}$$

For a particular value or threshold x_p with a non-exceedance probability *p*, there is a relationship:

$$T = \frac{1}{n(1-p)} \tag{3}$$

in which T is the *return period* or recurrence interval (year) that corresponds to the hydrological variable $X = x_p$ and *n* is the annual average number of occurrence of *X* during the period in which $F(x)$ is estimated. x_p is often called as *quantile* or T-year event. Note that if we deal with annual maximum data series, *n* = 1:

$$T = \frac{1}{1-p} \tag{4}$$

The 50-year event (*T*=50) has a non-exceedance probability *p* = 0.98. Likewise, *T* = 100 then *p* = 0.99 and *T* = 200 then *p* = 0.995.

Since $p = F(x_p)$, if we set *T* or *p*, we can obtain the quantile (T-year event) x_p by using the inverse function of *F*:

$$x_p = F^{-1}(p) \tag{5}$$

which gives the value of x_p corresponding to any particular value of *p* or *T*. Now the frequency of hydrological extreme value (annual maximum value) is related to frequency, return period or non-exceedance probability.

2.2 Probability distributions

There are many reasonable probability distributions (frequency analysis models) for modeling extreme events. Important families include the normal, extreme-value type I and type II, and Pearson type III (or Gamma) distributions. The probability density function (pdf) of the lognormal distribution with three parameters (LN3) is:

$$f(x) = \frac{1}{(x-c)\sigma_Y\sqrt{2\pi}} \exp\left[-\frac{1}{2}\left\{\frac{\ln(x-c)-\mu_Y}{\sigma_Y}\right\}^2\right] \tag{6}$$

in which *x* is a hydrological variable; a is lower bound parameter; μ_Y= the mean of *y* = $\ln(x-a)$; σ_Y= the standard deviation of *y*. Its cumulative distribution function (CDF) is in a from of the well-known standard normal distribution function Φ as:

$$F(x) = \Phi\left[\frac{y-\mu_Y}{\sigma_Y}\right] = \Phi[s] = \int_{-\infty}^{s} \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{1}{2}t^2\right)dt \tag{7}$$

in which $s = \frac{y-\mu_Y}{\sigma_Y}$ is the reduced (or standardized) variable of the original variable *y*.

The Gumbel distribution's CDF is:

$$F(x) = \exp\left[-\exp\left(-\frac{x-c}{a}\right)\right] \tag{8}$$

The generalized extreme-value (GEV) distribution has a CDF with a parameter $k \neq 0$:

$$F(x) = \exp\left[-\left(1-k\frac{x-c}{a}\right)^{1/k}\right] \tag{9}$$

If *k* = 0, the GEV distribution is equivalent to the Gumbel distribution shown as Eq. (8).

Another family Pearson type III (Gamma) distribution's CDF for $c \leq x < \infty$ is:

$$F(x) = G\left(\alpha, \frac{x-c}{\beta}\right) \Big/ \Gamma(\alpha) \tag{10}$$

where $\Gamma(\alpha)$ is the gamma function and $G(\alpha, x)$ is the “incomplete gamma function” defined as:

$$\Gamma(\alpha) = \int_0^\infty e^{-u} u^{\alpha-1} du, \ \ G(\alpha, x) = \int_0^x e^{-u} u^{\alpha-1} du \tag{11}$$

For a set of hydrologic extreme-value time series data (annual maxima data), hydrologists often try several distributions as these and select one to calculate the estimate of quantile (T-year event) x_p for a concerned non-exceedance probability *p*, which corresponds to the return period *T* by using Eq. (5).

Note that we can obtain analytical solutions for the extreme-value theory such as the Gumbel and GEV distributions, respectively:

$$x_p = -a \ln[-\ln p] + c \tag{12}$$

and

$$x_p = -\frac{a}{k}\left[(-\ln p)^k - 1\right] + c \tag{13}$$

However, since the families of the normal and Pearson type III distributions cannot have such analytical solutions, they require computational solutions. Many statistical textbooks include the standard normal distribution table, which shows the computational solutions for this purpose.

2.3 Choice of distributions

If we had sufficient knowledge of a hydrologic phenomenon, one could derive its population probability distribution without depending on observed data. If we had sufficiently long record of the phenomenon, we could determine its frequency distribution precisely, as long as the distribution did not change over time. But how should we select a model that approximates the population distribution based on the limited knowledge and data generally available?

When fitting distributions to a set of hydrologic extreme-value time series data, we often find that several distributions achieve almost the same goodness of fit, but can generate different estimates of extreme quantiles. In this situation, how should we select the “best” distribution?

Takara and Takasao (1988ab, 1990b) propose an attractive model selection procedure with six steps:

Step 1: Evaluate homogeneity and independence of data;
Step 2: Enumerate several distributions as candidates for quantile estimation;
Step 3: Estimate parameters for each distribution;
Step 4: Screen the distributions to assess goodness of fit (excluding those distributions that give worse fit);
Step 5: Analyze the variability of quantile estimates for distributions that have not been excluded in the prior step, by using a resampling method such as the jackknife or the bootstrap; and
Step 6: Select a distribution that fits data well and exhibits the smallest variability for quantile estimators.

Conventional model selection procedures have generally used goodness-of-fit criteria and/or tests in **Step 4**, and not included **Step 5** and **Step 6**. When two or more distributions achieve almost the same good results in fitting, conventional procedures have not been able to decide which is most appropriate. The proposed procedure will select the final distribution in terms of both the goodness of fit and the stability of quantile estimators, which is preferable from a practical viewpoint. While it is not difficult to estimate quantile variability, quantile accuracy (which is really of greater concern) is more difficult to determine because the true distribution of the events and its associated parameters are unknown.

3 PARAMETER ESTIMATION METHODS

Step 3 in the frequency analysis requires estimation of parameters of distribution functions. Parameter estimation is to fit a probability distribution to a set of data. There are several philosophies upon which fitting methods (parameter estimation methods) can be based. The most common are methods of moments, probability weighted moments, quantiles, maximum likelihood, maximum entropy, and least squares. It is difficult to definitely say which is the most appropriate method for a particular model and dataset.

However, Monte Carlo experiments are able to evaluate fitting methods for a range of distributions. The experiments basically include the following procedure:

1. Assume a population distribution (including population parameters);
2. Generate many samples of a specified size, whose observations are drawn from the assumed population by using random number generation techniques;
3. Fit the distribution model to each sample by some method and obtain estimators of parameters and quantiles; and
4. Evaluate the fitting method in terms of accuracy of the estimators.

Table 1. Monte Carlo studies comparing various fitting methods (Takara and Stedinger, 1994)

P.D.F.	**Paper**	**Fitting methods (number: methods compared)**	**Sample size**
LN(2)	Stedinger (1980)	5: ML*, MoM(b), modified ML, MoM(u), MoM(W)	10, 25, 50
LN(3)	Stedinger (1980)	4: Qt(S)+MoM(real)*, Qt(S)+ML, MoM(W), MoM(u)	25, 50
	Hoshi *et al.* (1984)	7: Qt(S)+MoM(real)*, MoM(u), Qt(I)+MoM(real), ML, Qt(S)+MoM(log), Qt(I)+MoM(log), MoM(BR)	20, 40, 80
	Takeuchi & Tsuchiya (1988)	7: Qt(S)+MoM(real)*, MoM(u), Qt(S)+MoM(log), PWM, Qt(S)+PWM(log, real), Sx	10 - 100
	Takara & Takasao (1990a)	10: Qt(I)+MoM*, MoM(b)*, MoM(u), PWM, LS(H), ML, MoM(IT), MoM(BR), LS(W), Sx	10 - 500
PIII(3)	Hoshi & Leeyavanija (1986)	5: Qt(S)+MoM*, Qt(S)+ML*, Sx*, MoM(u, BR)	20, 40, 80
LPIII(3)	Hoshi & Leeyavanija (1986)	6: Qt(S)+MoM(real)*, MoM(real)*, Qt(S)+MoM(log), Sx, Qt(S)+ML, MoM(BR)	20, 40, 80
EV1(2)	Landwhr *et al.* (1979)	3: ML*, PWM, MoM(u)	5 - 999
	Lettenmaier & Burges (1982)	4: ML*, MoM(b), MoM(u), Gum	10 - 100
	Raynal & Salas (1986)	7: ML*, BLCOS, PWM, MoM(b, u), LS(W), MIR	9, 19, 49, 999
	Phien (1987)	4: ML*, ME, PWM, MoM(u)	30, 40, 50
	Takara *et al.* (1989)	6: ML*, ME, PWM, MoM(u), LS(H), LS(W)	10 - 1000
GEV(3)	Hosking *et al.* (1985)	3: PWM*, Sx, ML	15, 25, 50, 100
	Takara *et al.* (1989)	4: PWM*, ML, MoM(u), MoM(b)	10 - 1000

P.D.F. (probability distribution functions) considered are: **LN**, lognormal; **PIII**, Pearson type III; **LPIII**, log-Pearson type III; **EV1**, Gumbel; **GEV**, generalized extreme-value. The number of parameters is given in the parentheses after the abbreviation.

Fitting methods are: **MoM**, method of moments, where the abbreviations (b, u) for two-parameter distributions indicates that each uses the biased and unbiased variances, respectively, whereas the abbreviations (b, u, W, BR and IT) for three-parameter distributions indicate (b) the skew based on the biased variance, (u) the skew based on unbiased variance, (W) the standard deviation and skew corrected by Wallis *et al.* (1974), (BR) the skew corrected by Bobée and Robitaille (1975), and (IT) the skew corrected by Ishihara and Takase (1957), respectively ('log' and 'real' indicate that the variable is considered in log and real space, respectively); **PWM**, probability-weighted moments; **ML**, maximum likelihood; **Qt**, quantile method using lower-bound-parameter estimation proposed by Iwai (I) or Stedinger (S); **Sx**, sextile method; **LS**, least-squares (the H and W indicate the use of plotting formula by Hazen and Weibull, respectively); **Gum**, Gumbel's regression method; **BLCOS**, best linear combination of order statistics method; **MIR**, mode and interquartile range method; and **ME**, maximum entropy method.

Note: In the **Fitting methods** column, * indicates generally best. However, best methods may change with the population skew, the sample size, and the exceedance probability for quantiles.

Takara and Stedinger (1994) summarized Monte Carlo studies comparing and evaluating various fitting methods for the lognormal (LN), Pearson type III (PIII), log-Pearson type III (LPIII), Gumbel (EV1) and generalized extreme-value (GEV) distributions. Table 1 shows what distributions and methods were addressed, as well as sample sizes considered. Abbreviations for fitting methods are used for simplicity (e.g., ML, the method of maximum likelihood; MoM, the method of moments). See the explanation given below the table. The methods with asterisk in the column of fitting methods were generally best for estimating quantiles with small exceedance probabilities. This preference may depend on population skew and sample size.

From Table 1, we notice that:

1) The maximum likelihood (ML) method gave accurate quantile estimators for two-parameter distributions: LN(2), EV1(2).
2) For three-parameter distributions such as the LN(3), PIII(3), and LPIII(3), the quantile lower bound estimation method coupled with the methods of moments or maximum likelihood (Qt&MoM or Qt&ML) gives more accurate quantile estimates. For the detailed discussions about lower-bound estimators, see Takara and Stedinger (1994).
3) For GEV(3), the method of probability-weighted moments (PWM) is best. The PWM is now well know as the method of L-moments (Hosking and Wallis, 1997) and often used for GEV(3).

4 GOODNESS-OF-FIT CRITERIA

To screen candidate distributions, Takara and Takasao (1988, 1989) used four criteria SLSC, COR, AIC, MLL to evaluate goodness of fit of each distribution to a hydrological extreme-value dataset quantitatively. Suppose that S is the reduced or standardized variate for X: $S=g(X)$. For example, for the normal (Gaussian) distribution with the mean μ and standard deviation σ,

$$S = g(X) = \frac{X-\mu}{\sigma} \tag{14}$$

Let p be a non-exceedance probability. For p^*, a specific value of p, define s^*:

$$s^* = g(F^{-1}(p^*)) \tag{15}$$

Let $\{y_1,\cdots,y_N\}$ be the order statistics ($y_1 \le y_2 \le \cdots \le y_N$) for the original observations $\{x_1,\cdots,x_N\}$, and p_i be the non-exceedance probability assigned to y_i ($i=1,\cdots,N$). Note that here y_1 is the smallest value and N is the number of observations (sample size). Using the transformation function g, we obtain:

$$s_i = g(y_i) \tag{16}$$

and

$$r_i = g(F^{-1}(p_i)). \tag{17}$$

4.1 SLSC (standard least-squares criterion)

The SLSC was originally introduced for evaluating linearity of the data (order statistics) plotted on a probability paper (Takasao *et al.*, 1986). It is calculated as:

$$\text{SLSC} = \frac{\sqrt{\delta^2_{\min}}}{\left| s^*_{1-p} - s^*_{p} \right|} \tag{18}$$

in which s^*_p is a specific value of the reduced variate s corresponding to the non-exceedance probability p^*, and $\delta^2_{\min}$ is obtained by minimizing:

$$\delta^2 = \frac{1}{N}\sum_{i=1}^{N}(s_i - r_i)^2. \tag{19}$$

This minimization operation corresponds to the so-called least-squares method (or one of the graphical fitting methods using a probability paper) based on a plotting position formula expressed in a general form as:

$$p_i = \frac{i-\alpha}{N+1-2\alpha} \tag{20}$$

in which α is constant. Takasao *et al.* (1986) recommended the use of Hazen's formula (α=0.5) to give p_i. The denominator in Eq. (18) is introduced to standardize the square root of $\delta^2_{\min}$. Thus the SLSC can be used to compare goodness of fit across distributions. For the non-exceedance probability *p* in Eq. (18), *p*=0.99 is used because most hydrological samples have less than 100 observations. Smaller SLSC value imply better fits. The SLSC is useful for absolute goodness-of-fit evaluation as well as comparison (relative evaluation). When using the ML (maximum likelihood) method, for example, instead of the least-squares method, we substitute δ^2 obtained by the ML method in to Eq. (18).

Takasao et al. (1986) fitted five two-parameter distributions (the normal, lognormal, exponential, Gumbel and log-Gumbel distributions) by the least-squares methods to samples in the Lake Biwa basin: the monthly and yearly precipitation and inflows to the lake, and the annual maximum *m*-day precipitations (*m*=1, 2 and 3). They concluded that SLSC $\approx$0.02 corresponds to a good fit; if SLSC>0.03, other distributions should be tried. They also compared six plotting position formulae: the Weibull (α=0.0), Adamowski (α=0.25), Blom (α=0.375), Cunnane (α=0.4), Gringorten (α=0.44) and Hazen (α=0.5). For a number of 70-year datasets of annual maximum *m*-day precipitation (*m*=1, 2 and 3), Hazen's formula gave better quantile estimates than the other five formulae for the lognormal and Gumbel distributions. Here, "better" means "nearest to those (analytical solutions) obtained by the ML method", so that Hazen's formula is recommended.

Afterward, Tanaka and Takara (1999a) found out that SLSC<0.03 is too strict for river discharge extremes because river discharges have some multi-cross-section problem to estimate their values from the water stage information by using the stage-discharge (H-Q) relationships. They concluded that SLSC<0.04 is acceptable to river discharge frequency analysis, investigating the goodness of fit of various probability distributions in about 100 major river basins in Japan.

4.2 COR (correlation coefficient)

Another goodness-of-fit criterion is the correlation coefficient between the ordered statistics y_i and r_i:

$$\text{COR} = \frac{\sum_{i=1}^{N}(y_i-\bar{y})(r_i-\bar{r})}{\sqrt{\sum_{i=1}^{N}(y_i-\bar{y})^2\sum_{i=1}^{N}(r_i-\bar{r})^2}} \tag{21}$$

in which $\bar{y}$ and $\bar{r}$ are the means of *y* and *r*, respectively. Values of COR closer to unity correspond to better fits. Takara and Takasao (1989) showed that COR=0.995 corresponds to SLSC=0.02, and COR 0.990 to SLSC=0.03. This probability plot correlation coefficient test has been applied to the normal, lognormal and Gumbel distributions (Vogel, 1986), and the Pearson type III distribution (Vogel and McMartin, 1991).

4.3 MLL (maximum log-likelihood)

It is well known in statistics that for larger samples the ML method gives preferable estimates of parameters in terms of unbiasedness and efficiency. Let $f(x;\theta)$ be a probability density function corresponding to a cumulative distribution function $F(x;\theta)$, for the variate X, where θ is a parameter vector consisting of *k* parameters. Given a series of independent observations $\{x_1,\cdots,x_N\}$, the ML method selects the estimates $\hat{\theta}$ that maximizes the likelihood function:

$$L(x_1,\cdots,x_N;\theta) = \prod_{i=1}^{N} f(x_i;\theta) \tag{22}$$

Numerical optimization techniques are used for distributions having no simple analytical expressions for the ML estimators. For reasons of computational tractability, we usually maximize the log-likelihood functions obtained by taking logarithm of Eq. (22). The maximized log-likelihood is given by

$$\text{MLL} = \sum_{i=1}^{N} \log f(x_i; \hat{\theta}) \tag{23}$$

in which $\hat{\theta}$ is the maximum likelihood estimator of θ. When several distributions are fitted to a sample, the distribution that gives the greatest MLL value can be regarded as fitting the best to the sample.

The MLL is not only the maximum of the log-likelihood; it has some interpretation from a viewpoint of information theory. If the population (real) distribution is known, the Kullback-Leibler (KL) information is used as an evaluation criterion for the models that approximate the population distribution. In general, it is unknown; then the MLL can be used as an alternative criterion instead of the KL information (Sakamoto et al., 1983).

4.4 AIC (Akaike information criterion)

In general, distributions with three free parameters fit better than those having two free parameters. As the number of parameters increases, goodness of fit should appear to improve: the SLSC values decrease and the COR and MLL values increase. Consequently, as long as the SLSC, COR or MLL is used, distributions having more parameters tend to be evaluated as "better". In the evaluation of models, we must consider model simplicity as well as the goodness of fit. The AIC proposed by Akaike (1974) balances the number of parameters q and the quality of fit, using:

$$\text{AIC} = -2 \log \text{(maximum likelihood)} + 2q = \text{MLL} + 2q \tag{24}$$

As q increases, the second term of Eq. (24) increases, while the first term decreases because the goodness of fit becomes better (the MLL increases). Akaike (1974) suggests that the model that minimizes the AIC is best. The AIC has been applied to hydrology, for example, in determination of the optimal order of time series models (Hipel *et al.*, 1977) and in evaluation of rainfall-runoff models (Takasao *et al.*, 1984).

Table 2. Comparison of the goodness of fit for the annual maximum daily precipitation at Osaka, Japan for 92 years, 1889-1980 (Takara and Takasao, 1988)

P.D.F. (q)	SLSC	COR	MLL	AIC
Normal (2)	0.07937	0.9312	-450.15	904.30
Lognormal (2)	0.02996	0.9902	-434.91	873.83
Lognormal (3)	0.01666 ***	0.9970 ***	-432.82 ***	871.64 **
Pearson type III (2)	0.06116	0.9685	-438.17	880.34
Pearson type III (3)	0.03765	0.9685	-432.90 **	871.80 *
Log-Pearson type III (3)	0.01749 **	0.9967 **	-432.91 *	871.82
SQET (2)	0.02423	0.9932	-433.09	870.18 ***
Gumbel (2)	0.04769	0.9846	-434.41	872.83
GEV (3)	0.02124	0.9944	-433.17	872.34
Log-Gumbel (2)	0.03496	0.9895	-434.53	873.06
Log-Gumbel (3)	0.01858 *	0.9960 *	-433.17	872.34

The number of parameters q is given in the parentheses. *** indicates the best distribution for each criterion: ** and * the second and the third.

Table 2 shows the result of fitting various distributions to the annual maximum daily precipitations at Osaka, Japan by using the ML method. The SLSC, COR and MLL gave good rankings to distributions with three adjustable parameters such as Lognormal (3), Pearson type III (3), Log-Pearson type III (3), Log-Gumbel (3) distributions, while the best distribution for the AIC was the SQET (2) distribution proposed by Etoh *et al.* (1987) with only two parameters. Many applications of these criteria to various hydrological extreme-values indicate that several candidate models can achieve almost the same goodness of fit, which implies we need some other criterion for selecting a final model.

5 THE JACKKNIFE RESAMPLING FOR EVALUATING VARIATION OF QUANTILE ESTIMATES

The quantile estimates and their accuracy are very important for decision making in various water-related design and planning. Resampling methods such as the jackknife and the bootstrap are very useful tools for quantifying the variability of quantile estimates. The resampling methods correct the bias of the statistics (estimates) obtained from the original dataset and estimate the variance of the statistics, producing many datasets by repeatedly sampling a part of the data from the original set or by repeatedly drawing samples of the same size as the original set with replacement. Datasets produced in this way are relatively easily generated using computers. Examples of these resampling methods are provided by Bardsley (1977), Tung and Mays (1981), Cover and Unny (1986) and Potter and Lattenmaier (1990).

Analytical expressions are available for the variance of quantile estimators in many cases (Stedinger *et al.*, 1993). Namely, the variance is given by a function of the non-exceedance probability *p*, sample size *N*, and parameter vector θ:

$$\mathrm{Var}(\hat{x}_p) = \mathrm{func}(p, N, \theta) \tag{25}$$

For the normal distribution and some other two-parameter distributions, Eq. (25) provides accurate variances, while it provides approximate values that may be inaccurate for three-parameter distributions (Hoshi *et al.*, 1984). In addition, this kind of equation has not been developed for some distributions.

On the other hand, resampling methods can be used for any distributions. They directly provide the bias-corrected quantile estimators and their variance, as well as their empirical distributions from observed data. If there is good coincidence between the variance analytically provided by Eq. (25) and that obtained by resampling methods, then we can use either one; though this kind of research has not been done. Takara and Takasao (1988) proposed applying the jackknife method in hydrologic frequency analysis to quantifying the variability of quantile estimates obtained by candidate probability distribution functions (**Step 5** in the section *2.3* above).

Let $\psi(x_1,\cdots,x_N)$ be a statistic representing some characters of the population (parent dataset). It is obtained by using *N* data: $x_1,\cdots,x_N$. Then the jackknife algorithm is:

i) For a sample of size N, using all the data, we obtain the estimate of ψ:

$$\hat{\psi} = \psi(x_1,\cdots,x_N) \tag{26}$$

ii) Using *N*-1 data excluded the *i*-th datum ($i = 1,\cdots,N$), we obtain

$$\hat{\psi}_{(i)} = \psi(x_1,\cdots,x_{i-1},x_{i+1}\cdots,x_N) \tag{27}$$

iii) The average of $\hat{\psi}_{(i)}$ is given by

$$\hat{\psi}_{(\bullet)} = \frac{1}{N}\sum_{i=1}^{N}\psi_{(i)} \tag{28}$$

iv) Quennoulli's estimate of bias is

$$\mathrm{Bias} = (N-1)(\hat{\psi}_{(\bullet)} - \hat{\psi}) \tag{29}$$

Consequently, the jackknife estimate $\hat{\psi}_J$ corrected for bias is

$$\hat{\psi}_J = \hat{\psi} - \mathrm{Bias} = N\hat{\psi} - (N-1)\hat{\psi}_{(\bullet)} \tag{30}$$

The jackknife estimate of variance of the statistic ψ is

$$\hat{s}^2{}_J = \frac{N-1}{N}\sum_{i=1}^{N}(\hat{\psi}_{(i)} - \hat{\psi}_{(\bullet)})^2 \tag{31}$$

It is known that $\hat{\psi}_J$ has a bias of order N^{-2} (Efron, 1982).

Regarding the quantile x_p (*T*-year event) as the statistic ψ in the jackknife algorithm above, we can easily quantify the variability of quantile estimators. Namely, fitting a distribution to the sub-datasets constructed in the manner of resampling through a parameter estimation method (e.g., ML method), we obtain the quantile estimates corresponding to $\hat{\psi}_{(i)}$ in Eq. (27) and then obtain the jackknife estimate of the quantile and its variance given in Eqs. (30) and (31), respectively. The proposed procedure for selection of a quantile estimation method, in Step 5, compares the variability of the quantile estimate for each distribution obtained by this method.

Table 3 shows the estimates of three T-year events (T=50, 100 and 200) and the estimates of their standard errors obtained by the jackknife method for the models which were not excluded in the model screening step. Table 3 indicates that:

1) The SQET(2) distribution was ranked as the best for the dataset of annual maximum precipitation at Osaka in terms of the smallest variation of the quantile estimates.
2) The variations (standard errors) of the T-year precipitations for T=50, 100 and 200 for the SQET(2) distribution were about 6.2, 6.7 and 7.1 %, respectively.
3) Three-parameter distributions tend to give larger variability of quantile estimates. In general, three-parameter distributions fit each dataset well; but larger quantile variation is a penalty that results from such flexibility.

Table 3. The jackknife estimates and standard error for *T*-year daily precipitation (in mm) at Osaka, Japan for 92 years, 1889-1980 (Takara and Takasao, 1988)

P.D.F. (q)	***T*=50**	***T*=100**	***T*=200**
SQET (2)	180.46 [11.25]	203.56 [13.58]	227.85 [16.08]
Pearson type III (3)	172.82 [14.68]	189.34 [17.34]	205.51 [20.36]
Lognormal (3)	179.94 [17.83]	201.66 [23.69]	224.17 [30.44]
Log-Pearson type III (3)	181.99 [19.56]	205.69 [27.06]	230.87 [36.13]
Log-Gumbel (3)	182.95 [21.07]	207.62 [30.35]	233.86 [43.03]
GEV (3)	183.10 [21.19]	207.83 [30.42]	234.15 [42.13]

The value in the square brackets is the standard error obtained by the jackknife.

6 TREND ANALYSES OF HYDROLOGIC EXTREME EVENTS

Recently it is often said that there could be a trend that the extreme values are becoming bigger and more frequent. The hydrological phenomena are non-stationary rather than stationary. We checked this by using long-term record of precipitation at many places in Japan.

Hydrologists have been interested in trend testing for hydro-meteorological variables such as precipitation, temperature and streamflow. Helsel and Hirsch (1992) provided a comprehensive review of statistical approaches used for trend analysis of water resources time series. The Mann-Kendall test is often used in trend analysis as a non-parametric method. This test was originally derived by Mann (1945) and developed by Kendall (1975) subsequently. Hirsch et al. (1982) used this test in water quality trend test application.

6.1 Mann-Kendall test

For a time series of a hydrological variable $\{x_1, \cdots, x_N\}$, we obtain a statistic:

$$S = \sum_{i=1}^{N-1} \sum_{j=i+1}^{N} \mathrm{sgn}(x_j - x_i) \tag{32}$$

where

$$\mathrm{sgn}(x_j - x_i) = \begin{cases} 1 & (x_j - x_i > 0) \\ 0 & (x_j - x_i = 0) \\ -1 & (x_j - x_i < 0) \end{cases} \tag{33}$$

The mean and variance of S are given:

$$E[S] = 0 \quad , \quad \mathrm{var}[S] = \frac{N(N-1)(2N+5)}{18} - \sum_{j=1}^{k} \frac{t_j(t_j-1)(2t_j+5)}{18} \tag{34}$$

where N is the number of data, k is the number of combinations with the successive same values that are obtained when ascending sort is done, and t_j is the number of the successive same values.
Normalizing S, we obtain

$$Z = \begin{cases} \dfrac{S-1}{\sqrt{\mathrm{var}[S]}} & (S > 0) \\ 0 & (S = 0) \\ \dfrac{S+1}{\sqrt{\mathrm{var}[S]}} & (S < 0) \end{cases} \tag{35}$$

If we obtain Z>1.96, the time series concerned is regarded as an increasing trend with the confidence level of 95 percent; if Z<1.96 a decreasing trend with the confidence level of 95 percent. Note that the Mann-Kebdall test has a disadvantage that it cannot detect actual trend, if the time series is too short or if the derivative is too small.
Using annual maximum daily precipitation for the period of 1901-2006 at 51 observatories (106-year records for 50 observatories and a 101-year record for Naha, Okinawa and) operated by the Japan Meteorological Agency (JMA), Kobayashi (2009) analyzed their trend and found out that the time series of annual maximum daily precipitation of 6 observatories (Fukushima, Fushiki, Sakai, Nagasaki, Kumamoto, Kochi) has increasing trend, while 1 observatory (Akita) has decreasing trend.

6.2 Application to GCM outputs

The JMA and the Meteorological Research Institute have developed a super-high resolution (20km) global climate model (GCM) and carried out a numerical experiment of climate variation for the present (1979-2003), near future (2015-2039) and future (2075-2099) periods under some climate change scenario. Kobayashi *et al.* (2010) estimated the 100-year daily rainfalls for the three periods using the GCM outputs.

The Mann-Kendall test has been carried out for the confirmation of the steadiness of the annual maximum daily rainfall time series that is the output of the GCM with non-steady sea surface temperature boundary conditions. The test confirmed that the steady statistical analysis is applicable to each data set. Then, the 100-year daily rainfall of the GCM present climate is estimated by the GEV distribution with the L-moment parameter estimation method and compared with the 100-year daily rainfall at the 51 meteorological observatories in Japan as mentioned above. The difference of the 100-year rainfall between the present and the future periods are analyzed. The results indicated that the 100-year daily rainfall increases at 56% of GCM output nodes in Japan for the near future, while it increases at 65% of nodes for the future periods comparing with the present period. Further details will be presented at a lecture on December 3, 2009.

REFERENCES

Bobée, B. and Robitaille, R. (1975) Correction of bias in the estimation of the coefficient of skewness, *Water Resources Research*, **11(6)**, pp. 851-854.

Efron, B. (1979) Computers and the Theory of Statistics: Thinking the Unthinkable, *SIAM REVIEW*, Vol. 21, No. 4, pp. 460-480.

Efron, B. (1982) *Jackknife, the Bootstrap and Other Resampling Plans*, SIAM Monograph, No. 38, 92 pp.

Etoh, T., Murota, A. and Nakanishi, M. (1987) SQRT-exponential type distribution of maximum, in *Hydrologic Frequency Modeling*, (ed.) V.P. Singh, D. Reidel Pub. Co., Dordrecht, Holland, pp. 253-264.

Helsel, D.R., and Hirsch, R.M. (1992) *Statistical Methods in Water Resources*, Elsevier Science Publishing, New York, 522 pp.

Hirsch, R.M., Slack, J.R. and Smith, R.A. (1982) Techniques of trend analysis for monthly water quality data, *Water Resources Research*, **18(1)**, pp. 107-121.

Hoshi, K. and Leeyavanija, U. (1986) A new approach to parameter estimations of gamma-type distributions, *Journal of Hydroscience and Hydraulic Engineering*, JSCE, **4(2)**, pp. 79-95.

Hoshi, K., Stedinger, J.R. and Burges, S.J. (1984) Estimation of log-normal quantiles---Monte Carlo results and first-order approximations, *Journal of Hydrology*, 71, pp. 1-30.

Hosking, J.R.M. and Wallis, J.R. (1997) *Regional Frequency Analysis: An approach based on L-moments*, Cambridge University Press, Cambridge, UK.

Hosking, J.R.M., Wallis, J.R. and Wood, E.F. (1985): Estimation of the generalized extreme-value distribution by the method of probability-weighted moments, *Technometrics*, 27(3), pp. 251-261.

Ishihara, T. and Takase, N. (1957) The logarithmic-normal distribution and its solution besed on moment method, *Trans. JSCE*, 47, pp. 18-23 (in Japanese).

Iwai, S. (1949) The Methods of Probable Flood Estimation and Their Application to Japanese Rivers, *Bulletin of Mathematical Statistics*, Vol. 2, No. 3, pp. 21-36, pp. 64-83 and 102 (in Japanese).

Jenkinson, A.F. (1955) The Frequency Distribution of the Annual Maximum (or Minimum) Values of Meteorological Elements, *Quart. Jour. Roy. Meteor. Soc.*, Vol. 81.

Kendall, M.G. (1975) *Rank Correlation Measures*, Charles Griffin, London.

Kobayashi, K. (2009) About climate change impacts on annual maximum precipitations in Japan (presented at a seminar in DPRI, Kyoto University (personal communication).

Kobayashi, K., Takara, K. and Nakakita, E. (2010) Estimation of the 100-year daily rainfalls in Japan projected by a GCM, *Annual Journal of Hydraulic Engineering*, JSCE, Vol. 54 (to be published in February 2010).

Landwehr, J.M, Matalas, N.C. and Wallis, J.R. (1979) Probability weighted moments compared with some traditional techniques in estimating Gumbel parameters and quantiles, Water Resources Research, **15(5)**, pp. 1055-1064.

Lettenmaier, D.P. and Burges, S.J. (1982) Gumbel's extreme value I distribution: a new look, *Journal of Hydraulic Div., Proc. ASCE*, 108, HY4, pp. 502-514.

Mann, H.B. (1945) Non-parametric tests against trend, *Econometrica*, **13**, MathSciNet, pp. 245-259.

Raynal, J.A. and Salas, J.D. (1986) Estimation procedures for the type-1 extreme value distribution, *Journal of Hydrology*, **87**, pp. 315-336.

Sakamoto, H., Ishiguro, M. and Kitagawa, G. (1983) *Information Statistics*, Kyoritsu, Tokyo, pp. 27-64.

Slade, J.J. (1936) An Asymmetric Probability Function, *Trans. ASCE*, Vol. 62.

Stedinger, J.R. (1980) Fitting log normal distributions to hydrologic data, *Water Resources Research*, **16(3)**, pp. 481-490.

Stedinger, J.R., Vogel, R.M. and Foufoula-Georgiou, E. (1993) Frequency analysis of extreme events, in *Handbook of Hydrology*, D.R. Maidment (ed.), McGraw-Hill, Chap. 18, 66 pp.

Takara, K., Imamoto, H., Hayashi, T., Nakakita, E., Ichikawa, Y., Hashino T. and Nakamura, Y. (1999) Storm and flood disaster in the Nakagawa basin in 1998, *Annual of Disas. Prev. Res. Inst.*, Kyoto Univ. No. 42 B-2, pp. 235-253 (in Japanese).

Takara, K. and Stedinger, J.R. (1994) Recent Japanese Contributions to Frequency Analysis and Quantile Lower Bound Estimators, *Stochastic and Statistical Methods in Hydrology and Environmental Engineering*, Vol. 1, pp. 217-234.

Takara, K. and Takasao, T. (1988a) Criteria for evaluating probability distribution models in Hydrologic frequency analysis, *Proc. JSCE*, 393/II-9, pp. 151-160 (in Japanese)

Takara, K. and Takasao, T. (1988b) Evaluation criteria for probability distribution models in hydrologic frequency analysis, *Preprint, Fifth IAHR International Symposium on Stochastic Hydraulics*, University of Birmingham, UK, August 1988, Paper A5, 10 pp.

Takara, K. and Takasao, T. (1989) Closure for Y. Goda's Discussion on "Criteria for evaluating probability distribution models in Hydrologic frequency analysis", *Proc. JSCE*, 405/II-11, pp. 267-272 (in Japanese).

Takara, K. and Takasao, T. (1990a) Comparison of parameter estimation methods for hydrologic frequency analysis models, *Proc. Hydraulic Engineering*, JSCE, 34, pp. 7-12 (in Japanese).

Takara, K. and Takasao, T. (1990b) Evaluation of hydrologic frequency analysis models based on quantile variability obtained by resampling methods, *Proc. Fifth International Conference on Urban Storm Drainage*, Suita, Osaka, Japan, 2, pp. 587-592.

Takara, K., Takasao, T. and Tomosugi, K. (1996) Possibility and Necessity of Paradigm Shift in Hydrologic Frequency Analysis, *Proc. of Int'l Conf. on Water Resources and Environment Research*, Kyoto, Japan, Vol. 1, pp. 435-442.

Takasao, T., Shiiba, M. and Takara, K. (1984) Evaluation of runoff models by an information criterion, *Annual of Disas. Prev. Res. Inst.*, Kyoto Univ. No. 27 B-2, pp. 275-290 (in Japanese).

Takasao, T., Takara, K. and Shimizu, A. (1986) A basic study on frequency analysis of hydrologic data in the Lake Biwa basin, *Annual of Disas. Prev. Res. Inst.*, Kyoto Univ. No. 29 B-2, pp. 157-171 (in Japanese).

Takeuchi, K. and Tsuchiya, K. (1988) On relative accuracy of PWM estimates of normal and 3-parameter lognormal distributions, Proc, JSCE, 393/II-9, pp. 103-112 (in Japanese).

Tanaka, S. and K. Takara (1999a) Goodness-of-fit and stability assessment in flood frequency analysis, *Annual Journal of Hydraulic Engineering*, JSCE, Vol. 43, pp. 127-132 (in Japanese).

Vogel, R.M. (1986) The probability plot correlation coefficiet test for the normal, lognormal, and Gumbel distributional hypothesis, *Water Resources Research*, **22(4)**, pp. 587-590 (with correction, *Water Resources Research*, **23(10)**, p. 2013).

Vogel, R.M. and McMartin, D.E. (1991) Probability plot correlation goodness-of-fit and skewness estimation procedures for the Pearson type III distribution, *Water Resources Research*, **27(12)**, pp. 3149-3158.

Wallis, J.R., Matalas, N.C. and Slack, J.R. (1974) Just a moment!, *Water Resources Research*, **10(2)**, pp. 211-219.